

# Adaptive Fault Tolerance in Automotive Vehicle using Control Allocation based on the Pseudo Inverse along the Null Space for Yaw Stabilization

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**Abstract**— Yaw instability of automotive vehicles occurs dangerous accidents particularly while driving on wet or icy surfaces. Considering wet or icy situations as faults, fault tolerant controllers are suitable to handle the control of automotive vehicles. In order to have yaw stability and increasing maneuverability and safety of faulty systems, using control allocation methods are good choices. This paper proposes a control allocation method based on the pseudo inverse along the null space of the control matrix (PAN) to establish lateral stabilization in automotive vehicle.

## I. INTRODUCTION

Using fault tolerant controllers are increasing in advanced control systems in order to make systems safer in faulty situations. There are two main fault tolerant methodologies that are called active and passive methods. The passive methods are based on fixed structure controllers that robustly considering bounds of uncertainty [1]. The active methods control the faulty systems by changing the parameters adaptively based on faults. Active methods divided into three main methods. Multiple model controllers are one of active fault tolerant controllers that change the controllers adaptively based on the faults occur in system [2]. Reconfigurable controllers that change the parameters of controllers based on faulty system are another group of active fault tolerant controllers [3-6].

Redundant actuators are useful in advanced industrial systems in order to increase safety, maneuverability and flexibility of systems while faults occur [7-9]. Control allocation methods which use this redundancy to manage control signals among redundant actuators are also active fault tolerant methods [10-12]. Modular structure of control allocation methods [13] allow us to design a module of constant controller for normal situations and a module which allocates signal controls among actuators adaptively in faulty situations.

There are many control allocation methodologies that the simplest one is the constrained least squares and its modifications [14, 15]. Daisy chaining is another method for managing control signals among redundant actuators that divide actuators into certain groups and the system uses these groups sequentially when needed [16]. Reducing energy consumption is an important advantage of this

method. Direct allocation is the common methods of control allocation which is based on the pseudo inverse and considers control signals in constraints [15, 17]. This method is condensed into a constrained optimization problem [18, 19]. The most common methods of control allocation are based on optimization problems. Linear programming, quadratic programming and nonlinear programming are different shapes of optimization methods which can solve control allocation problems using iterative numerical algorithms [18-22].

Yaw stabilization is one of the main issues of automotive control particularly in driving on wet or icy surfaces. Anti-lock braking systems (ABS) which was introduced more than thirty years ago was the primary attempt to control yaw movement. Traction control systems (TCS) also improve lateral stability and maximize the contact forces between tires and road during braking and acceleration. Electronic stability program (ESP) which control the yaw motion and prevent over and under steering was used since 1992 [23].

This paper proposes an adaptive fault tolerant controller using control allocation based on the pseudo inverse of the null space of the control matrix [26] for lateral stabilization of an automotive vehicle. Faulty situations for this system are considered as wet and icy driving surfaces.

The paper is organized as follows. Section II presents the problem. Model of the system, model of fault and the control allocation method (PAN) for faulty situations are introduced in this section. This is followed by section III that the automotive vehicle dynamic model and the yaw stabilizer controller is introduced. Section IV is created to illustrate the effect of proposed fault tolerant control allocation method for an automotive vehicle in faulty situations. Simulations and discussions are brought in this section. At the end, the section V contains conclusion and brief discussion about the method.

## II. PROBLEM STATEMENT

### A. System

Consider the plant described by the following state space equation:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

Where  $x \in R^n$  and  $u \in R^m$  are the system states and the control input. The matrix  $B$  is column rank deficient:

$$\text{rank}(B) = k < m, \forall x \in R^n \quad (2)$$

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The input signal  $u(t)$  belongs to a compact set  $\Omega$  that is defined as:

$$u(t) \in \Omega \equiv \{u \in R^m \mid u^- \leq u \leq u^+\} \quad (3)$$

Where the constraints are defined so that  $u^- \equiv [u_1^-, u_2^-, \dots, u_m^-]^T$  and  $u^+ \equiv [u_1^+, u_2^+, \dots, u_m^+]^T$  [12].

### B. Fault

It will be assumed that the system subject to faults can be written as [11]:

$$\dot{x}(t) = Ax(t) + Bu(t) - BK(t)u(t) \quad (4)$$

and by defining the effectiveness gain matrix  $K(t)$  as:

$$K(t) = \text{diag}(k_1(t), \dots, k_m(t)) \quad (5)$$

$$0 \leq k_i(t) \leq 1$$

The state space equation of system facing actuator faults are given as:

$$\dot{x}(t) = Ax(t) + BW(t)u(t) \quad (6)$$

$$W(t) = I - K(t)$$

If  $k_i = 0$ , then the  $i$ th actuator is working perfectly and for  $k_i > 0$ , the  $i$ th actuator is faulty and if  $k_i = 1$ , then the  $i$ th actuator has completely failed.

### C. Control Allocation (PAN)

Pseudo inverse based control allocation methods are simple and have low computational cost in comparison with optimization based methods. An important disadvantage of these methods is neglecting the actuator physical limitations. In this section, the pseudo inverse method is modified to consider actuators' limitations in faulty systems.

Using faulty model of the system in (6) and the modular structure in Fig. 1, the virtual control in faulty situation is as below:

$$v_f(t) = BW(t)u(t) \stackrel{BW(t)=B_f(t)}{\Rightarrow} v_f(t) = B_f(t)u(t) \quad (7)$$

There are many solutions for (7) that one of them is the pseudo inverse solution as below:

$$u_p = B_f^T (B_f B_f^T)^{-1} v_f \quad (8)$$

To modify the disadvantages of pseudo inverse, a correcting vector  $u_n$  is defined as below:

$$u = u_p - u_n \quad (9)$$

Substitute (9) in (7):

$$v_f = B_f u = B_f (u_p - u_n) = v_f - B_f u_n \quad (10)$$

It is desired to use a correction vector such that  $B_f u_n = 0$ , thus  $u_n$  should lie in the null space of  $B_f$ . It is desired to choose the best vector in the null space of  $B_f$ . To have a degree of freedom for choosing  $u_n$ , it can be written as:

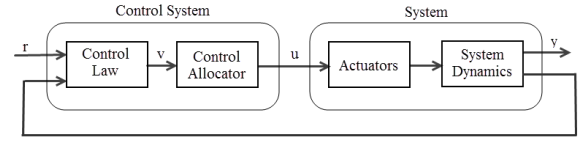


Fig. 1. Modular structure using control allocation.

$$u_n = N_f v_{free} \quad (11)$$

where  $N_f$  is the nullity matrix of  $B_f$  and  $v_{free}$  is a free vector that should be designed.

The modification of the pseudo inverse method is done by considering the constraints. The problem is that the constraints are unequal so the control signals should be normalized by defining the vector  $u_b$  as below:

$$u_{bi} = \begin{cases} u_i^+ & \text{if } u_{pi} > 0 \\ u_i^- & \text{if } u_{pi} < 0 \end{cases}, i = 1, \dots, m \quad (12)$$

Then, the normalization is as:

$$U^{-1}u = U^{-1}u_p - U^{-1}N_f v_{free} \quad (13)$$

And the matrix  $U$  is as below:

$$U = \begin{bmatrix} u_{b1} & 0 & 0 & 0 \\ 0 & u_{b2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & u_{bm} \end{bmatrix} \Rightarrow U^{-1}u_p = \begin{bmatrix} \frac{u_{p1}}{u_{b1}} \\ \frac{u_{p2}}{u_{b2}} \\ \vdots \\ \frac{u_{pm}}{u_{bm}} \end{bmatrix} \quad (14)$$

The main idea is to use correction vector in order to convey  $u_p$  to  $u$ , to put control signals in their constraints step by step. In each step, the normalized control signals which have maximum  $l_\infty$  norm are chosen as:

$$h = \max \left\{ \left| \frac{u_{pi}}{u_{bi}} \right| \mid i = 1, \dots, m \right\} = \max \left\{ \frac{u_{pi}}{u_{bi}} \mid i = 1, \dots, m \right\} \quad (15)$$

The elements of maximum  $l_\infty$  norm are saved in vector  $s$  which is defined as:

$$s = \left\{ i \mid \text{maximum} \left( \frac{u_{pi}}{u_{bi}} > 1 \right) \right\} \quad (16)$$

In each step, by assuming the aim vector ( $u_a$ ), try to decrease the control signals that have maximum  $l_\infty$  norm ( $u_{ps}$ ) as below:

$$u_a = u_{ps} - N_{fs} v_{free} \quad (17)$$

Where  $N_{fs}$  is formed of the rows of  $N_f$  matrix according to vector  $s$ . It is desired to convey to  $u_a$  in each step using minimum  $N_{fs} v_{free}$ . So  $v_{free}$  is obtained as below:

$$v_{free} = N_{fs}^T (N_{fs} N_{fs}^T)^{-1} (u_{ps} - u_a) \quad (18)$$

Define  $\Delta \equiv u_{ps} - u_a$  and substitute it in (9):

$$u = u_{ps} - N_f N_{fs}^T (N_{fs} N_{fs}^T)^{-1} \Delta \quad (19)$$

The below ratio is obvious for signals in  $s$  for each step:

$$\frac{u_{ps1}}{u_{bs1}} = \frac{u_{ps2}}{u_{bs2}} = \dots = \frac{u_{psk}}{u_{bsk}} \quad (20)$$

It is desired to decrease  $u_p$  such that its elements preserve the ratio (20). So  $\Delta$  can be computed using  $\bar{\Delta}$  instead of  $|\Delta_i|$  as below:

$$\Delta_i = \frac{u_{bsi}}{|u_{bs1}|} \bar{\Delta} \quad (21)$$

Using (19) and (21), the signal  $u$  obtained as:

$$u = u_p - u_r \bar{\Delta} \quad (22)$$

Where  $u_r$  is defined as below:

$$u_r \equiv N_f N_{fs}^T (N_{fs} N_{fs}^T)^{-1} \frac{u_{bs}}{|u_{bs1}|} \quad (23)$$

In order to equals the ratio of signals in  $s$  and signals that are not in  $s$  after modification, the ratio (24) should be satisfied:

$$\frac{u_{ps1} - u_{rs1} \bar{\Delta}}{u_{bs1}} = \frac{u_{pi} - u_{ri} \bar{\Delta}}{u_{bi}}, i=1, \dots, m, i \notin s \quad (24)$$

To have the least changes in every step, it is desired to choose the minimum value of  $\bar{\Delta}_i$ :

$$\bar{\Delta} = \min\{\bar{\Delta}_i | i=1, \dots, m, i \notin s\} \quad (25)$$

It is possible that during the reduction of  $u_{ps}$ , the sign of the numerator in (24) changes. To solve this problem, define  $d_i$  as below:

$$d_i = \text{sign}(u_{pi}) \times \text{sign}(u_{ri}), i=1, \dots, m, i \notin s \quad (26)$$

If  $d_i$  is positive, there is possibility that  $u_i$  changes its sign and consequently  $u_{bi}$  must change. Here, it is defined  $\bar{u}_{bi}$  and  $\bar{\Delta}_i$  as below:

$$\bar{u}_{bi} = \begin{cases} \bar{u}_i & \text{if } u_{pi} < 0 \\ \underline{u}_i & \text{if } u_{pi} > 0 \end{cases}, i=1, \dots, m \quad (27)$$

$$\bar{\Delta}_i = \frac{\frac{u_{ps1} - u_{pi}}{u_{bs1}} - \frac{u_{bi}}{\bar{u}_{bi}}}{\frac{u_{rs1} - u_{ri}}{u_{bs1}} - \frac{u_{bi}}{\bar{u}_{bi}}}, i=1, \dots, m, d_i = 1, i \notin s \quad (28)$$

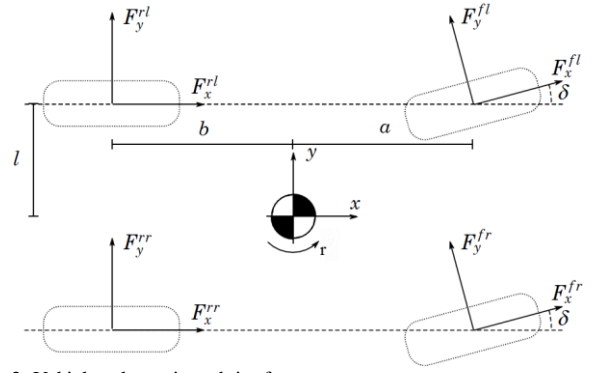


Fig. 2. Vehicle schematic and tire forces.

At the end, the minimum of  $\bar{\Delta}_i$  and  $\bar{\Delta}_i$  should be chosen as below:

$$\bar{\Delta}_i = \min(\bar{\Delta}_i, \bar{\Delta}_i), \bar{\Delta}_i \& \bar{\Delta}_i > 0 \quad (29)$$

If each one of  $\bar{\Delta}_i$  and  $\bar{\Delta}_i$  became negative, it should be ignored. At the last step, using the procedure like other steps, the normalized control signals goes under the limitations so as soon as  $u_{si}/u_{bsi} < 1$ , using  $\bar{\Delta}$  in a way that the desired  $u_a$  is equals to  $u_b$ :

$$\bar{\Delta} = |u_{ps1} - u_{bs1}| \quad (30)$$

The design vector  $v_{free}$  has  $m-n$  elements so it can just control  $m-n$  control signals in  $u_{ps}$ . If  $u_{ps}$  has more than  $m-n$  elements, the problem is infeasible and PAN is not efficient. The feasible method just has  $m-n$  steps so if after  $m-n$  steps, the signals do not lie in their constraints, it is an infeasible problem. To solve it, using the simple direct allocation method [19] which defines  $\alpha$  as below:

$$\alpha = \max(u/u_b) \quad (31)$$

Then decrease all the signals in a way that the maximum signal lies in its border:

$$u_{final} = u/\alpha \quad (32)$$

Notice that the signal  $u$  in (31) and (32) is the signal after  $m-n$  step of feasible solution in PAN.

### III. AUTOMOTIVE VEHICLE

#### A. Vehicle Model

The motion dynamics model of the vehicle [9] in Fig.2 with two front wheel steering and without roll dynamics is defined in (33) where the states are  $v$  as the velocity of the center of gravity (CG) of vehicle,  $\beta$  as the sideslip angle and  $r$  as the yaw rate.  $F_x$  is the total force at vehicle CG in  $x$  direction,  $F_y$  is the total force at vehicle CG in  $y$  direction and  $M_z$  is the Torque about the yaw axis. Also  $m$  is the vehicle mass and  $J_z$  is the vehicle moment of inertia about the yaw axis.

$$\begin{bmatrix} \dot{v} \\ \dot{\beta} \\ \dot{r} \end{bmatrix} = - \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{m} \cos(\beta) & \frac{1}{m} \sin(\beta) & 0 \\ -\frac{1}{mv} \sin(\beta) & \frac{1}{mv} \cos(\beta) & 0 \\ 0 & 0 & \frac{1}{J_z} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} \quad (33)$$

The individual tire forces in Fig. 2 are in relation with the generalized forces [27] as below:

$$F_x = F_x^{rl} + F_x^{rr} + (F_x^{fl} + F_x^{fr}) \cos(\delta) - (F_y^{fl} + F_y^{fr}) \sin(\delta) \quad (34)$$

$$F_y = F_y^{rl} + F_y^{rr} + (F_y^{fl} + F_y^{fr}) \cos(\delta) + (F_x^{fl} + F_x^{fr}) \sin(\delta) \quad (35)$$

$$\begin{aligned} M_z = & a(F_y^{fl} + F_y^{fr}) \cos(\delta) - b(F_y^{rl} + F_y^{rr}) \\ & + a(F_x^{fl} + F_x^{fr}) \sin(\delta) + l(F_x^{rl} + F_x^{rr}) \cos(\delta) \\ & + F_y^{fr} \sin(\delta) - F_x^{rr} - F_x^{fr} \cos(\delta) - F_y^{fl} \sin(\delta) \end{aligned} \quad (36)$$

$\delta$  is the steering angle and as can be seen in Fig. 2,  $\delta^{fl} = \delta^{fr} = \delta$  and  $\delta^{rl} = \delta^{rr} = 0$ . The relationship in (34-36) can be written as:

$$F_x = B_1 u \quad (37)$$

$$F_y = B_2 u \quad (38)$$

$$M_z = B_3 u \quad (39)$$

with  $u = [F_x^{fl} \ F_x^{fr} \ F_x^{rl} \ F_x^{rr} \ F_y^{fl} \ F_y^{fr} \ F_y^{rl} \ F_y^{rr}]^T$ .

To solve (37-39), it is appropriate to use the PAN method to allocating signals. By assuming equal lateral tire-road friction coefficient ( $\mu_y$ ) and equal longitudinal tire-road friction coefficient ( $\mu_x$ ) for four wheels, the total wheel forces are given as below:

$$F_x = -F_z \mu_x \quad (40)$$

$$F_y = F_z \mu_y \quad (41)$$

Where  $F_z$  is the total vertical force on ground and  $\mu_x, \mu_y$  are computed based on friction curves in Fig. 3 and  $\lambda_{xi}$  is wheel slip in longitudinal wheel direction and is defined by [28]:

$$\lambda_{xi} = \frac{v - \omega_i R}{v} \quad (42)$$

and  $\omega_i$  is the angular velocity of wheels,  $R$  is the radius of the wheels and  $\alpha_i$  is the wheel side slip angle.

### B. Yaw Stabilization

The main goal of this paper is yaw stabilization of an automotive vehicle. This leads to maneuverability of the vehicle in critical situations like driving on wet or icy surface. Using the controller (41) from [27] make the equilibrium  $r = r_{ref}$  globally stable in sense of Lyapunov.

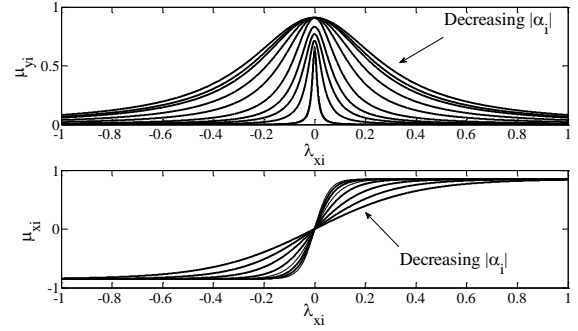


Fig. 3. Lateral and longitudinal friction coefficients as a function of longitudinal wheel slip.

$$M_z = -KJ_z (r - r_{ref}) \quad (43)$$

Also for safe driving and rollover prevention, the vehicle sideslip angle should be limited as below [29]:

$$\begin{cases} \text{for } v \leq 40 & |\beta| \leq 10^\circ - 7^\circ \frac{v^2}{(40m/s)^2} \\ \text{for } 40 < v \leq 50 & |\beta| \leq \end{cases} \quad (44)$$

## IV. SIMULATIONS

The model of the automotive vehicle presented in section III is used and the parameters of the model [30] is as follows:  $m = 1480kg$ ,  $J_z = 1950kg \cdot m^2$ ,  $a = 1.421m$ ,  $b = 1.029m$ ,  $l = 0.751m$ ,  $h = 0.42m$ ,  $g = 9.81m/s^2$ ,  $v_0 = 30m/s$ ,  $\beta_0 = 0.5rad$  and  $r_0 = 0.1rad/s$ . Assume that the front wheel steering angle is defined as Fig. 4 and the  $F_{zi}$ , the vertical force on ground from each wheel is defined in Fig. 5.

It is assumed that the vehicle in under faulty situations. The situations are modeled as the parameters of the effectiveness matrix [31] e.g. the effect of road friction reduced when the surface is wet or icy. So assume that the faulty situation is as below:

$$\begin{cases} \text{for } t \leq 22(s) & W(t) = \text{diag}(1,1,1) \\ \text{for } 22(s) < t \leq 42(s) & W(t) = \text{diag}(0.7,0.7,0.7) \\ \text{for } t > 42(s) & W(t) = \text{diag}(0.4,0.4,0.4) \end{cases} \quad (45)$$

Fig. 6 shows the states of the system. It can be seen that  $v$  and  $r$  are completely controlled and  $\beta$  leads in its constraints which is shown by dashed line. Fig. 7 shows the virtual control signals which produces by controller. Fig. 8 shows the allocated control signals in  $x$  direction to each tire and also Fig. 9 shows the allocated control signals in  $y$  direction to each tire. Also it can be seen in Fig. 8 and 9 that the PAN method considers control signals constraint and leads them in range of  $[-5000(Nm), 5000(Nm)]$ . Fig. 10 shows the parameters of the effectiveness matrix which are estimated using RLS algorithm. There can be seen some oscillations in Fig. 10 which is because of bad identifications while changes occur in system. But after that, it can be seen that the parameters are completely identified.

## V. CONCLUSION

This paper proposed a control allocation method for automotive vehicles in faulty situations. The control allocation uses actuator redundancies to manage forces among four tires in order to decrease the effect of fault and protect yaw stabilization. The control allocation method for both feasible and infeasible solutions was introduced and the automotive vehicle system and its controller were discussed. Simulation results were used to show the effectiveness of the proposed method.

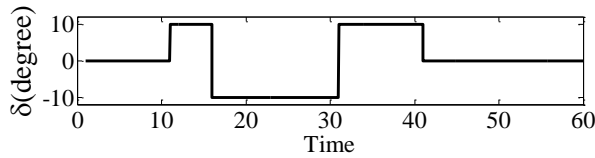


Fig. 4. Wheel steering angle.

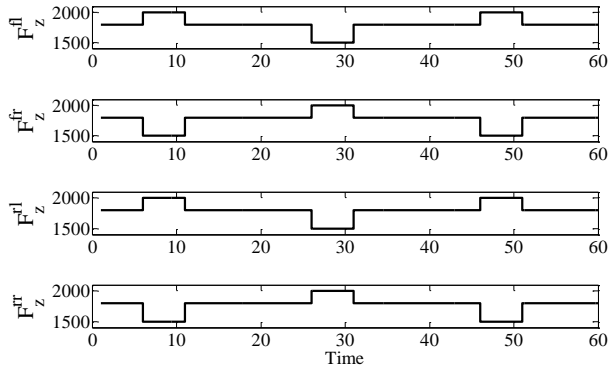


Fig. 5. Vertical force on ground from each wheel.

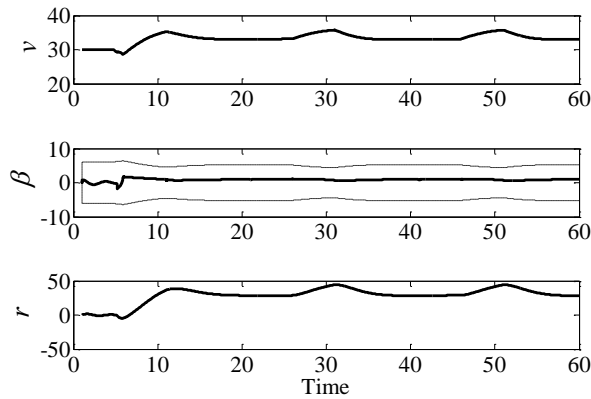


Fig. 6. System's states.

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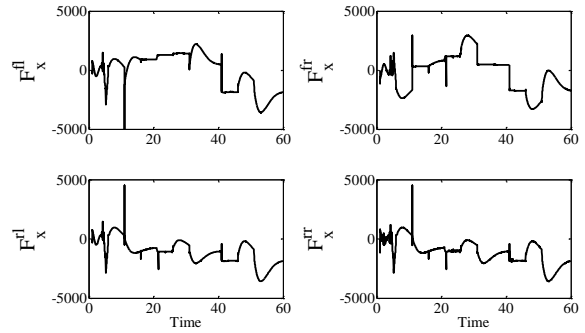


Fig. 8. Wheel forces in longitudinal direction.

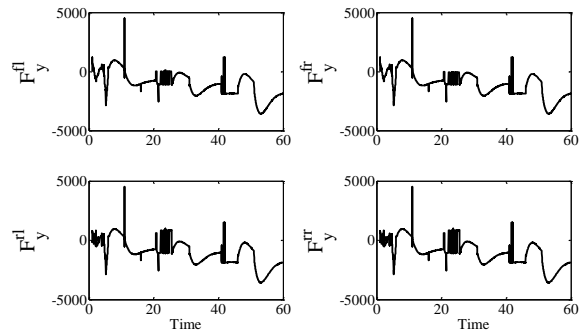


Fig. 9. Wheel forces in lateral direction.

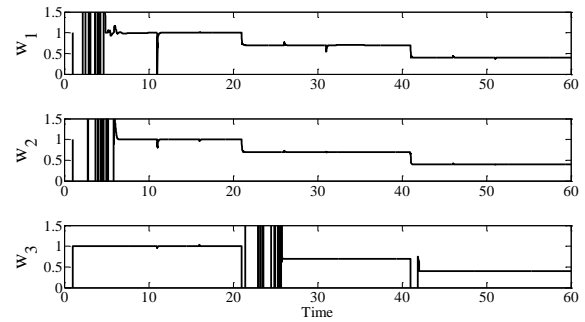


Fig. 10. Estimation of the effectiveness matrix parameters.

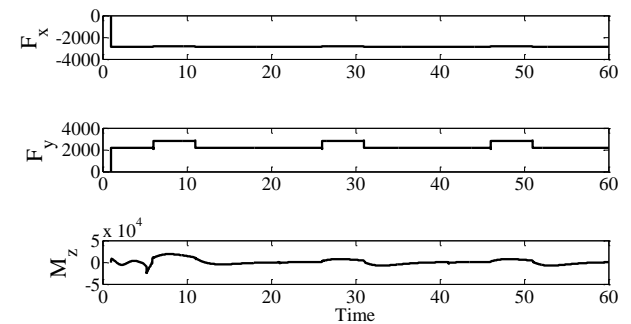


Fig. 7. Virtual control signals.

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